LISCOS : Scientific Highlights

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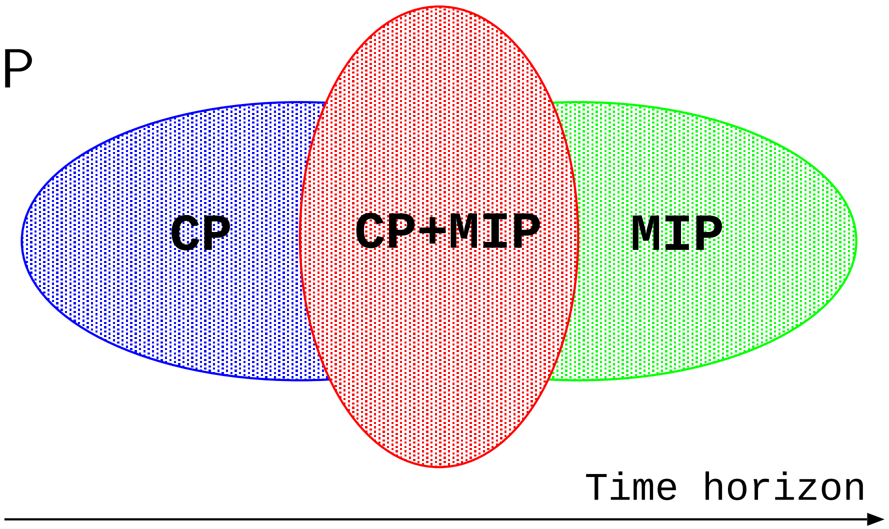
Bruxelles, 28 March 2003

MIP, CP, and their Combination

- Optimisation Technologies
  - Mixed Integer Programming : MIP
  - Finite Domain Constraint Programming : CP

- Supply chain optimisation
  - Long and mid-term planning : MIP
  - Short-term planning, scheduling : CP



⇝ supply chain optimisation
requires MIP, CP, and their combination

Practical Problem Solving

- Model building : Language

- Model solving : Algorithms

- (Model verification)

MIP vs. CP : Language

	MIP	CP
Variables	0-1	Finite domain
Constraints	Linear equations and inequalities	Arithmetic constraints Symbolic/global constraints

Example

- Variables : $x_1, \ldots, x_n \in \{0, \ldots, m-1\}$
- Constraint : Pairwise different values

- Integer programming: Only linear equations and inequalities

$$\begin{array}{lcccccccc} x_i \neq x_j & \iff & x_i & < & x_j & \vee & x_i & > & x_j \\ & \iff & x_i & \leq & x_j - 1 & \vee & x_i & \geq & x_j + 1 \end{array}$$

- Eliminating disjunction

$$x_i - x_j + 1 \leq m y_i, \quad x_j - x_i + 1 \leq m y_j, \quad y_i + y_j = 1,$$
$$y_i, y_j \in \{0,1\}, \quad 0 \leq x_i, x_j \leq m - 1,$$

- New variables: $z_{ik} = 1$ iff $x_i = k$, $\quad i = 1, \ldots, n, \ \ k = 0, \ldots, m-1$

$$z_{i0} + \cdots + z_{im-1} = 1, \quad z_{1k} + \cdots + z_{nk} \leq 1,$$

- Constraint programming $\rightsquigarrow$ symbolic constraint

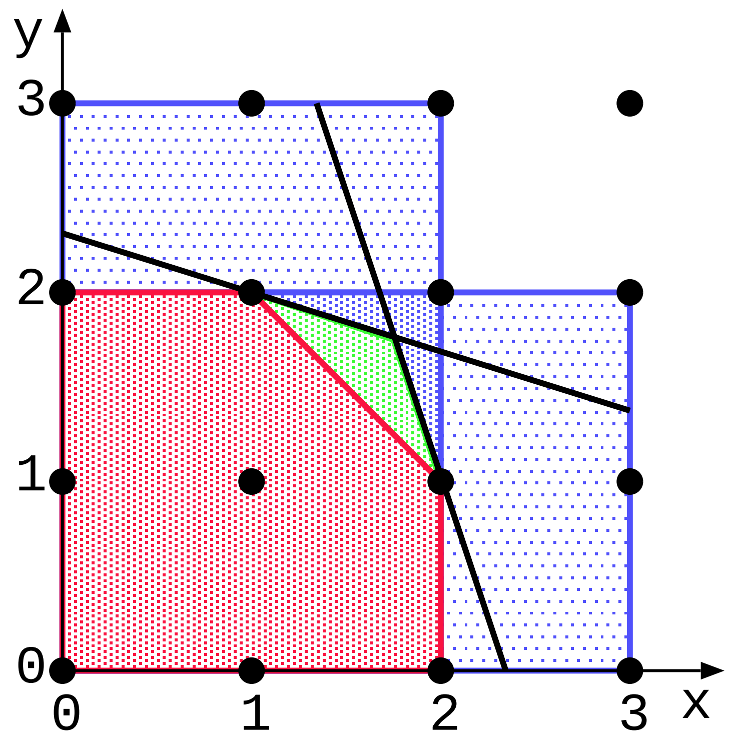
$$\texttt{alldifferent}(x_1, \ldots, x_n)$$

MIP vs. CP : Inference

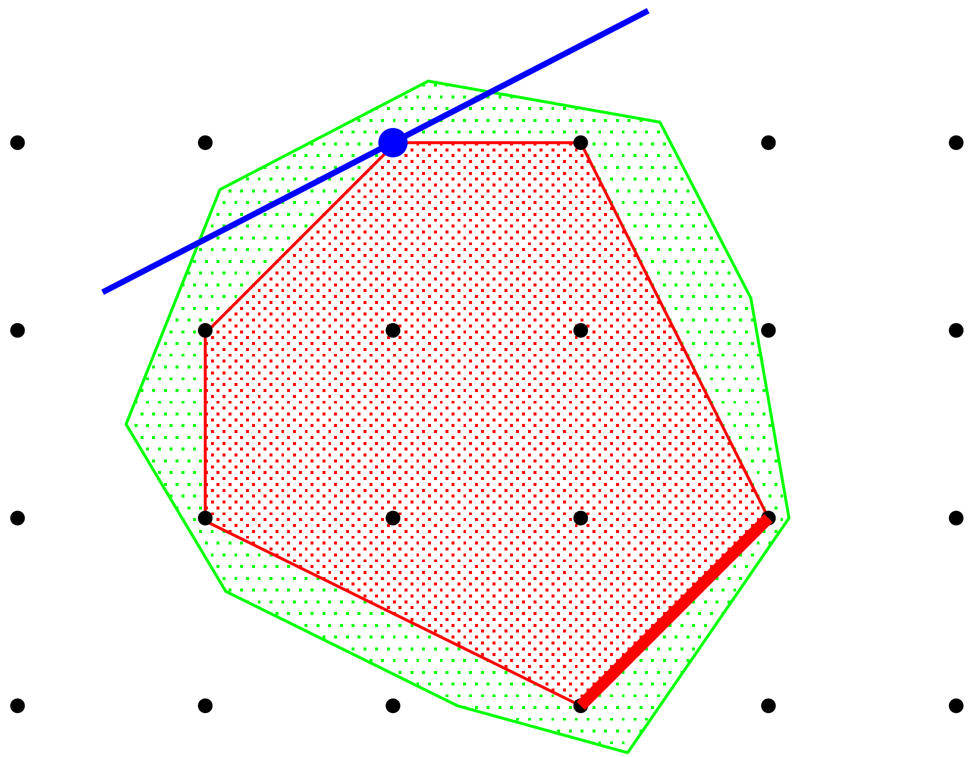
Linear arithmetic constraints

$$\begin{aligned} 3\,x + y &\leq 7, \\ 3\,y + x &\leq 7, \\ x + y &= z, \\ x, y &\in \{0, \ldots, 3\} \end{aligned}$$

CP	(Filtering) :	$x, y \leq 2,\ z \leq 4$
LP	(Linear programming) :	$x, y \leq 2,\ z \leq 3.5$
MIP	(Cutting plane) :	$x, y \leq 2,\ z \leq 3$



MIP : Tight Formulations



CP : Global Constraints

- $x_1, x_2, x_3 \in \{0, 1\}$
- pairwise different values
- Local consistency : 3 disequalities : $x_1 \neq x_2, x_1 \neq x_3, x_2 \neq x_3$
  $\leadsto x_1, x_2, x_3 \in \{0, 1\}$, i.e. no domain reduction is possible
- Global constraint : $\texttt{alldifferent}(x_1, x_2, x_3)$
  $\leadsto$ detects infeasibility (uses bipartite matching)

MIP vs. CP : Algorithms

	MIP	CP
Inference	Linear programming Cutting planes	Domain filtering Constraint propagation
Search	Branch-and-relax Branch-and-cut	Branch-and-bound
Bounds on the objective function	Two-sided	One-sided

LISCOS : Challenges

- Model building : MIP vs. CP
- Model solving : MIP vs. CP
- Problem decomposition
- Communication between solvers

$\rightsquigarrow$ generic modelling and solution approaches for MIP, CP, MIP+CP

LISCOS : Scientific Results

1. Modelling
   - Generic models
   - Classification
2. Mixed Integer programming
   - Reformulations
   - Valid inequalities, algorithms & heuristics
3. Constraint programming
   - Symbolic constraints for supply chain optimisation
   - Parametric search strategies
4. Combining MIP and CP
   - Cooperation schemes
   - Enhancing MIP branch-and-cut using CP

LISCOS : Software Architecture

- Mixed integer programming: XPRESS MP
- Constraint programming: CHIP
- Branch-and-cut for generic supply chain models: bc-gen
- Constraint programming for generic supply chain models: cp-gen
- Combined branch-and-cut + constraint programming: bc/cp-gen